Preparing for the
ACT
Mathematics &
Science Reasoning
SECOND EDITION

Dr. Robert D. Postman

AMSCO SCHOOL PUBLICATIONS, INC.
315 Hudson Street, New York, N.Y. 10013
Robert Postman is a college professor who is an expert in test preparation and subject-matter study. Dr. Postman holds a doctorate from Columbia University, where he received a full fellowship to pursue his graduate study. He is the author of over 30 books, which are found in schools and in bookstores throughout the United States. Dr. Postman has served as a consultant for many school districts and is widely recognized in numerous bibliographic publications, including *Who's Who in the World*, *Who's Who in America*, and *American Men and Women of Science*. He is also recognized for his work as a dean and department chair and for his faculty affiliation with Teachers College, Columbia University. An active participant in the community, he served on various boards, including special education boards, and as an elected member of the Board of Education.

**Reviewers**

Karen M. Brunner  
Okemos High School  
Okemos, MI  
Janie Mueller  
Cheyenne Mountain High School  
Colorado Springs, CO

Mona Busch  
Professional Development Institute  
Decatur, IL  
Jason D. Reissig  
Larkin High School  
Elgin, IL

Carol A. Goehring, NBCT  
Wekiva High School  
Orlando, FL  
Clayton P. Smith  
Larkin High School  
Elgin, IL

Dr. Yolanda Mendoza  
Miami Dade County Public Schools  
Miami, FL  
Dr. Maria J. Vlahos  
Barrington Community Unit School District 220  
Barrington, IL

TI calculator images reprinted by permission of the copyright owner, Texas Instruments

Please visit our Web site at: [www.amscopub.com](http://www.amscopub.com)  
When ordering this book, please specify:  
*either R 649 W or PREPARING FOR THE ACT: MATHEMATICS & SCIENCE REASONING, SECOND EDITION*


Copyright © 2011 by Amsco School Publications, Inc.  
No part of this book may be reproduced in any form without written permission from the publisher.  
Printed in the United States of America
Preparing for the ACT: Mathematics & Science Reasoning, Second Edition will help you get your highest possible score on these sections of the ACT. The result of a three-year effort, the book includes a thorough review of the subject matter, extensive practice exercises and problems, and effective strategies for taking the ACT. This book will improve your chances of being admitted to the school of your choice, and help you get the most out of college. This is an important opportunity, and I wish you well as you prepare to continue your education.

My special thanks go to my wife Betty Ann, who has been a constant source of support. I could not have completed this project without her. My children—Chad, Blaire, Ryan, and my grandson Quinn—have been an inspiration as I worked on this and other books over the years.

I would like to thank my editors, Pat Wilson and Uriel Avalos, at Amsco for their extraordinary efforts and commitment. I am also grateful to the teachers around the country who reviewed the manuscript and offered helpful suggestions. In addition, I am grateful to my son Ryan Postman, a mathematics teacher and an ACT tutor, and my wife Betty Ann, a mathematics teacher, who contributed significantly to the development of the book.

Special thanks go to the staff at ACT, who were very helpful as I worked on the manuscript. It was wonderful to speak with people who are truly interested in the students taking their test.

Robert D. Postman
# Contents

## Section I  Overview and Introduction

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Preparing for the ACT</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>The ACT Assessment</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>An ACT Website for Students</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Comparison of the ACT and the SAT</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Registering for the ACT</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Scoring</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>ACT Realities</td>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The ACT</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Mathematics Test Overview</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Science Reasoning Test Overview</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>English and Reading Tests Overview</td>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Test Strategies</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Test-Preparation Strategies</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>ACT Review Checklist</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Test-Taking Strategies</td>
<td>21</td>
</tr>
</tbody>
</table>

## Section II  Mathematics

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Mathematics Topic Inventory</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Introduction</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Topic Inventory</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Study Chart</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Calculators</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>Calculators and the ACT</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>Permitted Calculators</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>Prohibited Calculators</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Calculator Choices</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>Recognizing When Your Calculator Will Be Helpful</td>
<td>42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Pre-Algebra I</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Whole Numbers</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>Whole Number Computation</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Decimals</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>Decimal Computation</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Factors, Divisibility, and Primes</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>Square Roots, Exponents, and Scientific Notation</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>Fractions</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>Addition and Subtraction of Fractions and Mixed Numbers</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>Multiplication and Division of Fractions and Mixed Numbers</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>Positive and Negative Numbers</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Computation With Positive and Negative Numbers</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>Order of Operations</td>
<td>81</td>
</tr>
<tr>
<td></td>
<td>Cumulative ACT Practice—Pre-Algebra I</td>
<td>84</td>
</tr>
<tr>
<td>Chapter 12 Trigonometry</td>
<td>283</td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td>Right Triangle Trigonometry</td>
<td>283</td>
<td></td>
</tr>
<tr>
<td>Trigonometric Identities</td>
<td>288</td>
<td></td>
</tr>
<tr>
<td>Unit Circle Trigonometry</td>
<td>292</td>
<td></td>
</tr>
<tr>
<td>Graphs of Trigonometric Functions</td>
<td>297</td>
<td></td>
</tr>
<tr>
<td>Cumulative ACT Practice—Trigonometry</td>
<td>303</td>
<td></td>
</tr>
<tr>
<td>Trigonometry Subtest</td>
<td>304</td>
<td></td>
</tr>
<tr>
<td>Cumulative ACT Practice—Plane Geometry/Trigonometry</td>
<td>305</td>
<td></td>
</tr>
<tr>
<td>Plane Geometry/Trigonometry Subtest</td>
<td>308</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 13 Diagnostic Mathematics ACT: Model Mathematics ACT I</th>
<th>311</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checklist</td>
<td>323</td>
</tr>
<tr>
<td>Answers Explained</td>
<td>325</td>
</tr>
</tbody>
</table>

**Mathematics Answers** 334

---

**Section III  Model Mathematics Tests** 403

<table>
<thead>
<tr>
<th>Chapter 14 Model Mathematics ACT II</th>
<th>405</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answers and Explanations</td>
<td>417</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 15 Model Mathematics ACT III</th>
<th>425</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answers and Explanations</td>
<td>438</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 16 Model Mathematics ACT IV</th>
<th>445</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answers and Explanations</td>
<td>458</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 17 Scoring the Mathematics Tests</th>
<th>464</th>
</tr>
</thead>
</table>

---

**Section IV  Science Reasoning** 475

<table>
<thead>
<tr>
<th>Chapter 18 The Four-Step Approach to Taking the Science Reasoning Test</th>
<th>477</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passage and Question Types</td>
<td>477</td>
</tr>
<tr>
<td>Sample Passage With Answers Explained</td>
<td>479</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 19 Data Representation</th>
<th>484</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphs</td>
<td>484</td>
</tr>
<tr>
<td>Tables and Figures</td>
<td>488</td>
</tr>
<tr>
<td>Sample Passage With Answers Explained</td>
<td>490</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 20 Research Summaries</th>
<th>495</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyzing Research Design</td>
<td>495</td>
</tr>
<tr>
<td>Reading Research Summaries</td>
<td>496</td>
</tr>
<tr>
<td>Sample Passage With Answers Explained</td>
<td>497</td>
</tr>
</tbody>
</table>
Chapter 21 Conflicting Viewpoints 503
   The Science Vocabulary List 503
   Reading Conflicting Viewpoints Passages 506
   Sample Passage With Answers Explained 507

Chapter 22 Diagnostic Science Reasoning ACT:
   Model Science Reasoning ACT I 511
   Strategies for Passing the Science Reasoning ACT 511
   Checklist 527
   Answers Explained 528

Section V  ▪  Model Science Reasoning Tests 531

Chapter 23 Model Science Reasoning ACT II 533
   Answers and Explanations 548

Chapter 24 Model Science Reasoning ACT III 552
   Answers and Explanations 566

Chapter 25 Model Science Reasoning ACT IV 569
   Answers and Explanations 586

Chapter 26 Scoring the Science Reasoning Tests 589

Section VI  ▪  College Admission 591

Chapter 27 Getting into College 593
   So—You’re Thinking of Going to College 593
   Paying for College 601
   College and Financial Aid Glossary 603

Additional Answer Sheets 605

Index 613
Section I

Overview and Introduction

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 1</td>
<td>Preparing for the ACT</td>
<td>3</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>The ACT</td>
<td>8</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>Test Strategies</td>
<td>16</td>
</tr>
</tbody>
</table>
Chapter 1

Preparing for the ACT

The ACT Assessment

The ACT Assessment is a college admissions test. Colleges use ACT scores to help
determine which students will be admitted as freshmen or as transfer students. The ACT
consists of four separate multiple-choice tests: English, Reading, Mathematics, and
Science Reasoning.

Each test has a different number of items. The composite score is an average of the four
reported scores. ACT score reports show the composite score, the score for each test, and
subscores for groups of items that show achievement in particular areas.

You’ll find more detailed information about these tests, scores, test preparation and test-
taking strategies, subject reviews, and practice tests starting in the next chapter. This book
will lead you through the preparation you need to get your absolute best ACT score.

An ACT Website for Students

Check out the website ACT has just for students, at www.actstudents.org. This is our
favorite first stop for students who want to take the ACT.

You can complete your registration and receive your scores online through the site, and
you can hear from other students and their ACT experiences. This site also contains use-
ful information about financial aid and college planning.

We like that ACT is also on twitter.com, which will update you regularly about ACT
information. You can access the ACT twitter site from the ACT student web page.

Comparison of the ACT and the SAT

There are two national college admissions tests, the ACT from the American College
Testing Program and the SAT from the College Board and the Educational Testing Ser-
vice. Let me explain why you should take the ACT whether or not you take the SAT. The
ACT focuses more on achievement and is related to the high school curriculum. ACT test
makers are very clear about the material covered on the test and about the number of test
items devoted to each area. If those at the ACT say there will be six grammar and usage
items, that’s exactly how many there will be. Since items on the ACT are related to the
curriculum, you can effectively prepare for this test.

All the items on an ACT Assessment count toward your final score. On the SAT, one of
the sections is experimental and does not count.

The ACT Writing Test is optional. You must take the SAT Writing Test.
ACT score reporting and other policies are people-friendly. The ACT reports your scores quickly, which gives you plenty of time to decide about retaking the test. You can even decide which ACT scores will be reported to colleges, even after you have seen the scores.

The SAT penalizes you for incorrect answers. There is no incorrect answer penalty on the ACT, so you should guess whenever you can’t determine the correct answer.

### Registering for the ACT

You should register in advance for the ACT. You can register on the web at www.actstudent.org. ACT registration packets should be available in your high school. If you can’t find a registration packet, ask your guidance counselor, advisor, or teacher. You can also order a free registration packet on the ACT website or contact ACT directly:

ACT Registration
301 ACT Drive
P.O. Box 414
Iowa City, IA 52243-0414
(319) 337-1270 (Monday–Friday, 8:00–8:00 Central Time)
TDD (319) 337-1701 (for hearing-impaired persons calling from a TDD)

You can also call the ACT number to check on a late or delayed admission ticket, or to change your test date or test center. I called the ACT offices dozens of times while I worked on this book. Everyone I talked to was extremely helpful and pleasant. They want to help you, and you should feel very comfortable about calling.

The ACT has a website for students (www.actstudent.org). This website has complete information about the ACT, including registration information, test dates, and test sites. If you sign up for an ACT student web account, you have access to various services, for example, making changes to your registration. This site will be updated regularly. If you’re online, drop in to see what additional features or services have been added.

Regular ACT administrations occur on a Saturday in September, October, December, February, April, and June. Check the registration packets for test dates and registration deadlines. Registration ends about a month before the test date. Late registration for an additional fee ends about 15 days before the test date.

### When, Where, and What ACT to Take

You have to make four important registration decisions: (1) where to take the test, (2) in which school year to take the test, (3) when during the school year to take the test, and (4) what version to take: the regular ACT or the ACT Plus Writing.

You should take the ACT as close to home as possible. The test may even be given in your high school. The ACT is not given at every site on every test date. Check the registration packet to be sure the test is given at one of your preferred sites on the date you will take the test.
You should first take the ACT in your junior year. You can always take the test again in your senior year. Besides, application deadlines for many colleges and scholarship programs require you to take the ACT as a junior. Take the test toward the end of your junior year. I recommend the April test date. Since the ACT is closely tied to course content, junior-year classes will probably help. If you are taking the test in your senior year, take it early so the test scores are available to colleges.

Not all colleges require or recommend taking the writing test. Check with your college or guidance counselor first before signing up.

**Special Scoring Dates.** You can receive a copy of the test items, your scored answer sheet, and the correct answers (Test Information Release) if you take the test in December, April, or June at a national test center. This scoring information can be a valuable diagnostic tool. You can request this service on the registration form or when you receive your test scores in the mail. The test items, answers, and your answer sheet will be mailed to you eight to twelve weeks after the test date.

### Forms of Identification

You must bring an acceptable form of identification to the test center, or you probably won’t be able to take the test. Acceptable forms of identification include an up-to-date official photo ID or a picture from a school yearbook showing your first and last name. Unacceptable forms of identification include unofficial photo ID, learner’s permit or driver’s license without a photograph, a birth certificate, or a social security card. If you are not sure whether or not you have an acceptable ID, check the ACT student website or call the ACT ID Requirements Office at (319) 337-1510.

### Standby Registration

You may be able to register standby at an ACT test center. Needless to say, you should do everything you can to avoid standby registration. There is a good chance that there will be no room for you.

Show up at a center on test day with a valid ID, a checkbook, a credit card, and some hope. All those registered at that center are seated first. If there’s room, those registered at other centers are seated next. If there’s still room, you can fill out an application on the spot and take the test. There is an extra fee for this service.

### Alternate Testing Arrangements

If you live more than 50 miles from a test center, are confined at home or a hospital because of an illness, are in a correctional facility, or live in a country where there is no testing facility, you may be eligible for arranged testing. Check out the ACT student website for more information. Note that arranged testing is not available if you miss the test because of a schedule conflict.
If your religious beliefs prevent you from taking a Saturday test, you may take the ACT on a Sunday or a Monday. A limited number of sites in each state offer non-Saturday testing. If you live within 50 miles of those sites, you must take the test there on the date it is offered. Go to the ACT student website for more information.

If you have a diagnosed disability, you may qualify for special accommodations (such as permission to eat snacks if you are a diabetic), extra testing time, and/or special test dates. Please visit the ACT student website for details.

## Scoring

The maximum reported score for each test is 36, although each test has a different number of items. The composite score is an average of the four reported scores. The maximum composite score is 36. ACT score reports show the composite score, the score for each test, and subscores for groups of items that show achievement in particular areas. Many colleges use these subscores for placement. If you take the optional writing test, you will receive two additional scores: the combined English/writing score and the writing subscore. Your score on the writing test does not affect the composite score.

**Score Reporting.** Those at the ACT treat your scores as though they were your property. That means you decide who sees your scores, which scores they see, and when they see them. I discuss this score-reporting policy as a part of the overall testing strategy later in this section.

You can even take the ACT several times until you get a score you like and have only that score sent to colleges. Do not list the colleges for which you think a particular score is needed or required. Wait for the ACT to report the scores to you. Then decide whether and where to send the scores. I prefer this option, particularly if you know that a college may require a minimum score for admission. Consult with your guidance counselor to see if this is a good strategy for your case.

## ACT Realities

You take the ACT because it is required for college admissions or because it will help you get admitted to a college of your choice.

Tests can be unfair. A lucky guesser may occasionally do better on a multiple-choice test than someone who knows the material. Someone who knows the answers may get a lower score because he or she mismarks the answer sheet. Students who are sick the day of the test may do more poorly than they would have otherwise.
Some students may get a higher score than they have any right to expect. Other students may get a lower score than they need and deserve to receive. Students who know strategies for taking multiple-choice tests will often do better than students who don’t know these strategies. You’ve got to make the best of it and get your highest score. This book will show you how.

It’s Just People. The ACT is designed and written by people who have their own personal strengths and weaknesses. They are not perfect and neither is the test they create.

Consultants throughout the country make recommendations to the ACT test designers about the content that should be included on the test. These recommendations are based on the consultants’ knowledge of the subject matter and on the topics currently taught in American high schools.

The final list of topics is sent to test writers who actually prepare the test items. The test writers may be full-time employees of the American College Testing Program or they may be freelance writers from all over the country. The writers submit the items to the ACT, where the items are reviewed and edited. Then each item is reviewed further and tried out. The items that pass this review process are used on an ACT. Each item is used only once in its original form, but some items are revised and used in other ACTs.
## Study Chart

<table>
<thead>
<tr>
<th>STUDY TOPIC</th>
<th>QUESTION NUMBERS</th>
<th>PAGES TO STUDY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-Algebra I (Chapter 6)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ Whole Numbers</td>
<td>1, 2, 3, 4</td>
<td>44–48</td>
</tr>
<tr>
<td>□ Whole Number Computation</td>
<td>5, 6, 7, 8</td>
<td>48–51</td>
</tr>
<tr>
<td>□ Decimals</td>
<td>9, 10</td>
<td>52–55</td>
</tr>
<tr>
<td>□ Decimal Computation</td>
<td>11, 12, 13, 14</td>
<td>55–58</td>
</tr>
<tr>
<td>□ Factors, Divisibility, and Primes</td>
<td>15, 16</td>
<td>58–61</td>
</tr>
<tr>
<td>□ Square Roots, Exponents, and Scientific Notation</td>
<td>17, 18</td>
<td>61–64</td>
</tr>
<tr>
<td>□ Fractions</td>
<td>19, 20</td>
<td>64–67</td>
</tr>
<tr>
<td>□ Addition and Subtraction of Fractions and Mixed Numbers</td>
<td>21, 22</td>
<td>68–72</td>
</tr>
<tr>
<td>□ Multiplication and Division of Fractions and Mixed Numbers</td>
<td>23, 24</td>
<td>72–75</td>
</tr>
<tr>
<td>□ Positive and Negative Numbers</td>
<td>25, 26</td>
<td>75–77</td>
</tr>
<tr>
<td>□ Computation With Positive and Negative Numbers</td>
<td>27, 28, 29, 30</td>
<td>77–81</td>
</tr>
<tr>
<td>□ Order of Operations</td>
<td>31</td>
<td>81–83</td>
</tr>
<tr>
<td><strong>Pre-Algebra II (Chapter 7)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ Percent</td>
<td>32, 33</td>
<td>86–89</td>
</tr>
<tr>
<td>□ Percent Problems</td>
<td>34, 35</td>
<td>89–94</td>
</tr>
<tr>
<td>□ Ratio and Proportion</td>
<td>36, 37</td>
<td>94–97</td>
</tr>
<tr>
<td>□ Statistics—Mean, Median, and Mode</td>
<td>38, 39, 40</td>
<td>98–101</td>
</tr>
<tr>
<td>□ Data Collection, Representation, and Interpretation</td>
<td>41</td>
<td>101–106</td>
</tr>
<tr>
<td>□ Probability</td>
<td>42</td>
<td>107–110</td>
</tr>
<tr>
<td>□ Elementary Counting Techniques</td>
<td>43</td>
<td>110–114</td>
</tr>
<tr>
<td>□ Writing Linear Expressions and Equations</td>
<td>44</td>
<td>114–117</td>
</tr>
<tr>
<td>□ Solving Linear Equations</td>
<td>45</td>
<td>117–121</td>
</tr>
<tr>
<td><strong>Elementary Algebra (Chapter 8)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ Evaluating Formulas and Expressions</td>
<td>46, 47</td>
<td>125–128</td>
</tr>
<tr>
<td>□ Exponents and Radicals</td>
<td>48, 49</td>
<td>129–131</td>
</tr>
<tr>
<td>□ Operations with Radicals</td>
<td>50</td>
<td>132–134</td>
</tr>
<tr>
<td>□ Polynomials</td>
<td>51</td>
<td>135</td>
</tr>
<tr>
<td>□ Operations on Polynomials</td>
<td>52, 53</td>
<td>135–140</td>
</tr>
<tr>
<td>□ Factoring Polynomials</td>
<td>54, 55</td>
<td>141–144</td>
</tr>
<tr>
<td>□ Quadratic Equations</td>
<td>56</td>
<td>144–148</td>
</tr>
<tr>
<td><strong>Intermediate Algebra (Chapter 9)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>□ Solving Inequalities</td>
<td>57, 58</td>
<td>157–161</td>
</tr>
<tr>
<td>□ Absolute Value Equations and Inequalities</td>
<td>59, 60</td>
<td>161–163</td>
</tr>
<tr>
<td>□ Solving Systems of Linear Equations</td>
<td>61, 62</td>
<td>163–167</td>
</tr>
<tr>
<td>□ Rational and Radical Expressions</td>
<td>63</td>
<td>167–170</td>
</tr>
<tr>
<td>□ Solving Quadratic Equations</td>
<td>64</td>
<td>170–173</td>
</tr>
<tr>
<td>□ Solving Quadratic Inequalities</td>
<td>65</td>
<td>173–175</td>
</tr>
<tr>
<td>□ Complex Numbers</td>
<td>66, 67</td>
<td>175–177</td>
</tr>
<tr>
<td>□ Patterns, Sequences, and Modeling</td>
<td>68</td>
<td>178–180</td>
</tr>
<tr>
<td>□ Matrices</td>
<td>69, 70</td>
<td>180–185</td>
</tr>
<tr>
<td>STUDY TOPIC</td>
<td>QUESTION NUMBERS</td>
<td>PAGES TO STUDY</td>
</tr>
<tr>
<td>-------------</td>
<td>-------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Coordinate Geometry (Chapter 10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>☐ Graphing Inequalities on a Number Line</td>
<td>71, 72</td>
<td>188–190</td>
</tr>
<tr>
<td>☐ Graphing Equations on the Coordinate Plane</td>
<td>73</td>
<td>191–199</td>
</tr>
<tr>
<td>☐ Distance and Midpoint Formulas</td>
<td>74, 75</td>
<td>199–202</td>
</tr>
<tr>
<td>☐ Graphing Systems of Inequalities on the Coordinate Plane</td>
<td>76, 77</td>
<td>202–208</td>
</tr>
<tr>
<td>☐ Graphing Conic Sections</td>
<td>78, 79</td>
<td>208–215</td>
</tr>
<tr>
<td>Plane Geometry (Chapter 11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>☐ Basic Elements of Plane Geometry</td>
<td>80, 81</td>
<td>224–227</td>
</tr>
<tr>
<td>☐ Angles</td>
<td>82, 83, 84, 85</td>
<td>227–231</td>
</tr>
<tr>
<td>☐ Quadrilaterals</td>
<td>86</td>
<td>232–235</td>
</tr>
<tr>
<td>☐ General Properties of Triangles</td>
<td>87</td>
<td>235–238</td>
</tr>
<tr>
<td>☐ Right Triangles</td>
<td>88</td>
<td>238–243</td>
</tr>
<tr>
<td>☐ Similar Triangles</td>
<td>89</td>
<td>244–248</td>
</tr>
<tr>
<td>☐ Concept of Proof and Proof Techniques</td>
<td>90</td>
<td>249–254</td>
</tr>
<tr>
<td>☐ Circles</td>
<td>91, 92</td>
<td>255–260</td>
</tr>
<tr>
<td>☐ Transformations in the Plane</td>
<td>93</td>
<td>260–267</td>
</tr>
<tr>
<td>☐ Geometric Formulas</td>
<td>94</td>
<td>267–271</td>
</tr>
<tr>
<td>☐ Geometry in Three Dimensions</td>
<td>95</td>
<td>271–277</td>
</tr>
<tr>
<td>Trigonometry (Chapter 12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>☐ Right Triangle Trigonometry</td>
<td>96, 97</td>
<td>283–287</td>
</tr>
<tr>
<td>☐ Trigonometric Identities</td>
<td>98</td>
<td>288–291</td>
</tr>
<tr>
<td>☐ Unit Circle Trigonometry</td>
<td>99</td>
<td>292–296</td>
</tr>
<tr>
<td>☐ Graphs of Trigonometric Functions</td>
<td>100</td>
<td>297–301</td>
</tr>
</tbody>
</table>
Solving Inequalities

Solving inequalities is like solving equations, except that multiplying or dividing by a negative number reverses the inequality sign.

Multiplying or Dividing an Inequality
by a Negative Number

**Examples**

1. Multiply both sides of \(-4x \geq 7\) by \(-1\).
   \[-4x \geq 7 \rightarrow 4x \leq -7\]

2. Multiply both sides of \(-\frac{x}{6} < -4\) by \(-6\).
   \[-\frac{x}{6} < -4 \rightarrow x > 24\]

3. Divide both sides of \(9 \leq -3x\) by \(-3\).
   \[9 \leq -3x \rightarrow -3 \geq x\]

4. Divide both sides of \(-7x + 42 \leq y\) by \(-7\).
   \[-7x + 42 \leq y \rightarrow x - 6 \geq \frac{-y}{7}\]

Reverse the inequality sign when you multiply or divide both sides of the inequality by a negative number.
Solving Inequalities

To solve an inequality, follow the same steps as for solving equations.

Using Addition or Subtraction

**EXAMPLES**

1. Solve. \( x + 15 < 23 \)

\[
\begin{align*}
x + 15 &< 23 \\
-15 &\quad -15 \\
x &< \frac{8}{1}
\end{align*}
\]

To check, choose a number for \( x \) that is close to but less than 8.

Substitute 7 for \( x \) in the inequality. \( 7 + 15 < 23 \)

\( 22 < 23 \checkmark \)

2. Solve. \( -19 + x \geq 12 \)

\[
\begin{align*}
-19 + x &\geq 12 \\
+19 &\quad +19 \\
x &\geq \frac{31}{1}
\end{align*}
\]

*Check:* Since \( x \) is greater than or equal to 31, choose two values for \( x \).

Substitute 31 for \( x \). Substitute 32 for \( x \).

\( -19 + 31 \geq 12 \quad -19 + 32 \geq 12 \)

\( 12 \geq 12 \checkmark \quad 13 \geq 12 \checkmark \)

Using Multiplication or Division

**EXAMPLES**

1. Solve. \( -\frac{y}{8} > 7 \)

Multiply by -8. \( -\frac{y}{8}(-8) > 7(-8) \)

Reverse the inequality. \( y < -56 \)

To check, substitute -57 for \( y \) in the original inequality.

\[
\begin{align*}
-\left(\frac{-57}{8}\right) &> 7 \\
-(-7.125) &> 7 \\
7.125 &> 7 \checkmark 
\end{align*}
\]
2. Solve. \(-8k \leq 70\)

Divide by \(-8\).
\[
\frac{-8k}{-8} \leq \frac{70}{-8}
\]

Reverse the inequality.
\[
k \geq -8 \cdot \frac{3}{4}
\]

Substitute \(-8 \cdot \frac{3}{4}\) for \(k\).

\[
-8\left(-8 \cdot \frac{3}{4}\right) \leq 70
\]

\[
-8(-8) \leq 70
\]

\[
-8\left(-\frac{35}{4}\right) \leq 70
\]

\[
64 \leq 70 \checkmark
\]

\[
70 \leq 70 \checkmark
\]

You may have to use more than one operation to solve some inequalities.

### EXAMPLES

1. Solve. \(-5t - 8.5 < 54\)

Add 8.5.
\[
-5t - 8.5 < 54
\]

Divide by \(-5\).
\[
\frac{-5t}{-5} < \frac{62.5}{-5}
\]

Reverse the inequality.
\[
t > -12.5
\]

2. Solve. \(k + 8 + 3.4 > -28\)

Subtract 3.4.
\[
k + 8 > -31.4
\]

Multiply by 8.
\[
k > -251.2
\]

Do not reverse the inequality.

3. Solve. \(-3x - 4 \geq 5 + x\)

Subtract \(x\).
\[
-x
\]

\[
-4x - 4 \geq 5
\]

Add 4.
\[
+4
\]

Divide by \(-4\).
\[
\frac{-4x}{-4} \geq \frac{9}{-4}
\]

Reverse the inequality.
\[
x \leq -2.25
\]
MODEL ACT PROBLEMS

1. What is the solution set to the inequality $2x + 5 \geq 13$?
   
   A. $x \leq 9$
   B. $x \geq 9$
   C. $x \leq 4$
   D. $x \geq 4$
   E. $x \leq -4$

   SOLUTION
   
   
   Subtract 5.
   
   Divide by 2.
   
   The correct answer is D.

2. What is the smallest number in the solution set to the inequality $-4x - 5 \leq -2x + 9$?
   
   F. $-14$
   G. $-7$
   H. $0$
   J. $7$
   K. $14$

   SOLUTION
   
   Add 5.
   
   Add 2x.
   
   Divide by 2.
   
   The correct answer is G.

Practice

Solve each inequality.

1. $x + 9 > 13$
2. $-13 + x \leq 22$
3. $-3y \leq 7$
4. $-\left(\frac{y}{6}\right) > 11$
5. $-4t + 6 \leq 9$
6. $\frac{k}{8} - 2.4 > 12$
7. $-5x + 6 \leq 2x - 8$
8. $7 + 4t < 3t - 2$
9. $2x + 5 < 4$
10. $-3x > 9$
11. $2x - 7 \geq 12$
12. $7x - 3 < 3x + 1$
13. $-2x + 9 \leq 13$
14. $16x - 7 \approx 6x + 2$
15. $-4x + 9 \geq 27$
16. $-3x + 7 > 11x - 2$
17. $-x - 13 > 2$
18. $27x - 5 < 35x + 3$
19. $9x - 5 \leq 6x + 2$
20. $x + 3 < 2x - 1$

(Answers on page 359)

ACT-TYPE PROBLEMS

1. What is the solution set for the inequality $4x \geq -12$?
   
   A. $x \geq -4$
   B. $x \leq -3$
   C. $x \geq -3$
   D. $x \geq 3$
   E. $x \leq 4$

2. What is the solution set to the inequality $-3x - 7 < 20$?
   
   F. $x > 13$
   G. $x < 13$
   H. $x < 9$
   J. $x > 9$
   K. $x > -9$
3. What is the solution set to the inequality 
   \(-4x + 17 \leq -3\)?
   A. \(x \leq -5\)
   B. \(x \geq -4\)
   C. \(x \leq 4\)
   D. \(x \geq 5\)
   E. \(x = 5\)

4. Which of the following is not in the solution set to 
   the inequality 
   \(-5x + 3 > -2x - 12\)?
   F. 5
   G. 4
   H. 3
   J. 2
   K. 1

(Answers on page 360)

### Absolute Value Equations and Inequalities

When solving absolute value equalities and inequalities you must consider two possibilities.

For example: If \(|x| = 7\), then \(x = 7\) or \(x = -7\).

To solve absolute value equations and inequalities you must solve for each case.

Case 1: The value is positive. Drop the absolute value and solve.

Case 2: The value is negative. Drop the absolute value. Use a minus sign to make the 
expression from inside the absolute value negative and solve.

#### EXAMPLES

1. \(|x - 8| = 5\)
   Case 1: \(x - 8\) is positive \(x - 8 = 5\) \(x = 13\)
   Case 2: \(x - 8\) is negative \(-(x - 8) = 5\) \(-x + 8 = 5\) \(-x = -3\) \(x = 3\)
   \(x = 3\) or \(x = 13\)
   Check: \(|13 - 8| = |5| = 5\)
   \(|3 - 8| = |-5| = 5\)

2. \(|x + 4| < 7\)
   Case 1: \(x + 4 < 7\) \(x < 3\)
   Case 2: \(-(x + 4) < 7\) \(-x - 4 < 7\) \(-x < 11\) \(x > -11\)
   Therefore \(-11 < x < 3\).
   Check: Check a sample of the values between \(-11\) and 3. Each value makes the 
original inequality correct.
Solve.

1. \( \frac{3}{4}x + \frac{14}{28} = 14 \)
2. \( \frac{4}{6}x - \frac{1}{4} = 9 \)
3. \( \frac{3}{5}x + 3 = 11 \)
4. \( \frac{5}{2}x - 12 \geq 13 \)
5. \( \frac{3}{2}x + 8 \geq 12 \)
6. \( \frac{3}{4}x - 3 = 12 \)
7. \( \frac{6}{5}x + 5 > 9 \)
8. \( \frac{1}{2}x - 5 \leq 2 \)
9. \( \frac{3}{2}x - 1 < 5 \)
10. \( \frac{2}{3}x + 4 \leq 6 \)
11. \( |x - 3| < 1 \)
12. \( |2x + 7| = 9 \)
13. \( |7x - 3| \geq 7 \)
14. \( |5x + 4| \leq 4 \)
15. \( |4x - 9| > 3 \)
16. \( |3x + 3| = 4 \)
17. \( |8x - 5| \leq 9 \)
18. \( |2x + 3| < 12 \)
19. \( |5x - 5| \geq 7 \)
20. \( |x + 1| > 14 \)

(Answers on page 360)

ACT-TYPE PROBLEMS

1. 16 and -10 are the solutions to which one of the following equations?
   A. \( |x - 2| = 14 \)
   B. \( |x - 2| = 8 \)
   C. \( |x + 2| = 12 \)
   D. \( |x - 3| = 13 \)
   E. \( |x + 3| = 13 \)

2. Which of the following choices makes the inequality \(|2x - 9| < 5\) false?
   F. 2
   G. 3
   H. 4
   J. 5
   K. 6

3. Solve the inequality \(|7x - 5| \geq 9\).
   A. \(-4 \leq x \leq 2\)
   B. \(x \leq \frac{4}{7} \) or \( x \geq 2 \)
   C. \(x \leq -\frac{4}{7} \) or \( x \geq 2 \)
   D. \(x \leq -\frac{4}{7} \) or \( x \geq -2 \)
   E. \(-\frac{4}{7} \leq x \leq 2 \)

4. \(x = 6\) is the complete solution set to which of the following equations?
   F. \(2x = 12\)
   G. \(|x - 3| = 3\)
   H. \(|2x - 2| = 10\)
   J. \(-x = 6\)
   K. \(|x + 6| = 12\)
5. What is the product of the solutions to the equation $|2x - 3| = 15$?

A. 45  
B. 9  
C. -6  
D. -54  
E. -81

(Answers on page 360)

Solving Systems of Linear Equations

A linear equation is any equation in the form $ax + by = c$ ($a \neq 0$, $b \neq 0$). The solution to a linear equation is an ordered pair $(x, y)$ that makes the equation true.

A system of linear equations is two or more linear equations that can be solved together. The solution to a system of linear equations must be the solution for all of the equations in the system.

To solve a system of equations, sometimes you can add or subtract the equations to eliminate one of the variables. Other times you will have to change an equation so that when you add or subtract, one of the terms is eliminated.

**Examples**

1. Solve. $3x + 5y = 16$

   $-3x + 3y = 8$

   This one is easy. Add the two equations.

   \[ \begin{align*}
   3x + 5y &= 16 \\
   -3x + 3y &= 8 \\
   \hline
   8y &= 24 \\
   y &= 3
   \end{align*} \]

   Substitute 3 for $y$ in one of the equations.

   $3x + 5(3) = 16$

   Solve for $x$.

   \[ \begin{align*}
   3x + 15 &= 16 \\
   3x &= 1 \\
   x &= \frac{1}{3}
   \end{align*} \]

   The solution to the system is $x = \frac{1}{3}$ and $y = 3$.

   The solution as an ordered pair is \( \left( \frac{1}{3}, 3 \right) \).

**Calculator Tip**

Graphing calculators can be used to graph and solve systems of linear equations.
2. Solve. \(-4x - 4y = 8\)
\[2x + 7y = 15\]

Multiply the bottom equation by 2. \[2(2x + 7y = 15) \rightarrow 4x + 14y = 30\]

Add the equations. \[\begin{align*}
-4x - 4y &= 8 \\
4x + 14y &= 30
\end{align*}\]

Solve for \(y\). \[10y = 38 \quad y = 3.8\]

Substitute 3.8 for \(y\) in one of the equations. \[-4x - 4(3.8) = 8\]

Solve for \(x\). \[\begin{align*}
-4x &= 23.2 \\
4x &= -23.2 \\
x &= -5.8
\end{align*}\]

The solution is \(x = -5.8\) and \(y = 3.8\).

The solution as an ordered pair is \((-5.8, 3.8)\).

3. Solve. \(-8y + 5x + 12 = 2\)
\[6x + 12y = 6\]

Rewrite the equations in linear form. \[\begin{align*}
5x - 8y &= -10 \\
6x + 12y &= 6
\end{align*}\]

Multiply the top equation by 1.5. \[1.5(5x - 8y = -10) \rightarrow 7.5x - 12y = -15\]

Add the equations. \[\begin{align*}
7.5x - 12y &= -15 \\
6x + 12y &= 6
\end{align*}\]

Solve for \(x\). \[\begin{align*}
13.5x &= -9 \\
x &= \frac{-2}{3}
\end{align*}\]

Substitute \(-\frac{2}{3}\) for \(x\) in one of the equations. \[6\left(-\frac{2}{3}\right) + 12y = 6\]

Solve for \(y\). \[\begin{align*}
-4 + 12y &= 6 \\
12y &= 10 \\
y &= \frac{5}{6}
\end{align*}\]

The solution is \(x = \frac{-2}{3}\) and \(y = \frac{5}{6}\).

The solution as an ordered pair is \((-\frac{2}{3}, \frac{5}{6})\).
1. What is the solution to the system of linear equations $2x + 5y = 10$ and $2x + 3y = 2$?

A. $(-5, -4)$  
B. $(-5, 4)$  
C. $(-4, -5)$  
D. $(4, -5)$  
E. $(5, 4)$

**SOLUTION**

Subtract one equation from the other.

\[
\begin{align*}
2x + 5y &= 10 \\
-2x - 3y &= -2
\end{align*}
\]

Solve for $y$.

\[
\begin{align*}
2y &= 8 \\
y &= 4
\end{align*}
\]

Substitute $4$ for $y$ in one of the equations.

\[
\begin{align*}
2x + 5(4) &= 10 \\
2x &= -10 \\
x &= -5
\end{align*}
\]

The solution is $x = -5$ and $y = 4$.

The solution as an ordered pair is $(-5, 4)$.

The correct answer is B.

2. What is the sum of the solutions to the following system of linear equations?

\[
\begin{align*}
6x - 5y &= 15 \\
-3x + 2y &= 10
\end{align*}
\]

F. $-61 \frac{2}{3}$  
G. $-35$  
H. $8 \frac{1}{3}$  
J. $25$  
K. $26 \frac{2}{3}$

**SOLUTION**

Multiply both sides of the second equation by 2.  

\[
2(-3x + 2y = 10) \rightarrow -6x + 4y = 20
\]

Add the equations.

\[
\begin{align*}
6x - 5y &= 15 \\
-6x + 4y &= 20 \\
-2y &= 35 \\
y &= -35
\end{align*}
\]

Substitute $-35$ for $y$ in one of the equations.  

\[
-3x + 2(-35) = 10
\]

Solve for $x$.

\[
\begin{align*}
-3x - 70 &= 10 \\
-3x &= 80 \\
x &= -\frac{80}{3} = -26 \frac{2}{3}
\end{align*}
\]

The solutions are $x = -26 \frac{2}{3}$ and $y = -35$.

Find the sum of the solutions.

\[-26 \frac{2}{3} + (-35) = -61 \frac{2}{3}\]

The correct answer is F.
Practice

Solve the system of equations.

1. \[ 4x + 5y = 13 \]
   \[ 4x + 3y = 9 \]
2. \[ 3x - 2y = 6 \]
3. \[ -5y + 3x = \frac{8}{4} \]
   \[ 2y + 12x + 13 = 26 \]
4. \[ 2x + 5y = 6 \]
   \[ 2x + 4y = 5 \]
5. \[ 3x + 5y = 7 \]
6. \[ 4x - 9y = 8 \]
7. \[ 2x + 4y = 9 \]
   \[ 3x - 4y = 8 \]
8. \[ x + 3y = 5 \]
   \[ 2x + 4y = 6 \]
9. \[ 4x + 5y = 8 \]
   \[ 6x + 8y = 7 \]
10. \[ 5x + 12y = 13 \]
    \[ 3x + 4y = 5 \]
11. \[ 12x + 8y = 2 \]
    \[ 4x + 6y = 10 \]
12. \[ -6x + 7y = -13 \]
    \[ -12x + 8y = 4 \]
13. \[ x - 5y = 10 \]
   \[ -2x + 3y = 8 \]
14. \[ -12x + 8y = -5 \]
   \[ -4x + 6y = -5 \]
15. \[ -4x + 9y = 19 \]
   \[ -6x + 11y = -20 \]
16. \[ -2x + 5y = 6 \]
   \[ -3x + 4y = 9 \]
17. \[ 9x + 7y = 5 \]
   \[ 8x + 6y = 4 \]
18. \[ -16x + 7y = 5 \]
   \[ 17x - 8y = 2 \]

19. Holly has 15 dimes and nickels worth $1.05. How many of each type of coin does Holly have?

20. Suresh studied 8 hours for his final exams in math and science. He studied 1.5 hours longer for his math final than for his science final. How many hours did he study for each final?

21. Tia went to a sale at a media store. She bought 8 videos and CDs for $92. If each video cost $16 and each CD cost $10, how many of each did she buy?

(Answers on page 360)

ACT-TYPE PROBLEMS

1. What is the solution to the following system of linear equations?
   \[ 2x + 5y = 8 \]
   \[ 2x + 4y = 7 \]

   A. \((1.5,1)\)
   B. \((1.1.5)\)
   C. \((1,-1.5)\)
   D. \((-1.5,1)\)
   E. \((-1.5,-1)\)

2. What is the solution to the following system of linear equations?
   \[ 3x + 5y = 8 \]
   \[ -3x + 5y = 8 \]

   F. \((1.6,0)\)
   G. \((0.1.6)\)
   H. \((0.1.4)\)
   J. \((0,-1.6)\)
   K. \((-1.4,0)\)
3. What is the product of the solutions to the following system of linear equations?

\[ \begin{align*}
7x + 10y &= 12 \\
5x + 5y &= 6
\end{align*} \]

A. \(-1.2\)  
B. \(-1\)  
C. \(0\)  
D. \(1\)  
E. \(1.2\)

4. You are to find two numbers. When you double the first and triple the second, their sum is 1. When you triple the first and multiply the second by 5, the sum is 2. What are the two numbers?

F. \((1,0.5)\)  
G. \((1,-1)\)  
H. \((-0.5,1)\)  
J. \((-1,1)\)  
K. \((-1,-1)\)

(Answers on page 360)

5. There are two paths. In the morning, 6 people walked the first path and 12 people walked the second path. The total distance these people walked was 8 miles. In the afternoon, 9 people walked the first path and 4 people walked the second path. The total distance people walked in the afternoon was 5 miles. How many miles long is each path?

A. \((\frac{1}{2} \text{ mile}, \frac{1}{3} \text{ mile})\)  
B. \((\frac{1}{3} \text{ mile}, \frac{1}{2} \text{ mile})\)  
C. \((3 \text{ miles}, 2 \text{ miles})\)  
D. \((2 \text{ miles}, 3 \text{ miles})\)  
E. \((5 \text{ miles}, 8 \text{ miles})\)

(Rational and Radical Expressions)

Simplifying Expressions

To simplify rational and radical expressions, you may need to use some or all of these equalities.

- \(\sqrt[n]{x}\) means the \(n\)th root of \(x\).
- \(\sqrt[n]{x^n} = x\)
- \(x^{-(a)} = \frac{1}{x^a}\)
- \(\sqrt{x} \cdot \sqrt{x} = x\)

When the bases are the same, use these equalities to multiply and divide exponents.

- \(x^b \cdot x^a = x^{b+a}\)
- \(x^b \div x^a = x^{b-a}\)

Undefined Expressions

An expression is considered undefined when its denominator is equal to zero, or any time there is division by zero. Otherwise, the expression is defined.

- \(\frac{4\sqrt{3}}{x}\) is defined except when \(x = 0\).
- \(\frac{x^{\frac{1}{3}}}{3^x}\) is defined for all values of \(x\).
- \(\frac{4(2x + 8)}{x - 8}\) is defined for all values of \(x\) except \(x = 8\).
1. Simplify. \( \frac{11x}{\sqrt{3x - 8}} \)

\[
\frac{11x}{\sqrt{3x - 8}} = \frac{11x}{\sqrt{3x - 8}} \cdot \frac{\sqrt{3x - 8}}{\sqrt{3x - 8}}
\]

Multiply numerator and denominator by \( \sqrt{3x - 8} \). This removes the radical from the denominator.

\[
= \frac{11x\sqrt{3x - 8}}{3x - 8}
\]

2. Simplify. \( \frac{1}{x^{(-\frac{2}{3})}} + \frac{\sqrt{x^2}}{x^{(-\frac{1}{3})}} \)

\[
\frac{1}{x^{(-\frac{2}{3})}} + \frac{\sqrt{x^2}}{x^{(-\frac{1}{3})}} = x^{(\frac{2}{3})} + x^{(\frac{1}{3})} \cdot \sqrt{x^2}
\]

Use \( a^{-a} = \frac{1}{a^a} \) \( \left( \frac{1}{x^{(\frac{2}{3})}} = x^{(\frac{2}{3})} \right) \)

\[
= x^{(\frac{2}{3})} + x^{(\frac{1}{3})} \cdot \sqrt{x^2}
\]

Use \( \sqrt{x^2} = x^{(\frac{2}{3})} \) \( \left( x^{(\frac{2}{3})} \right) \)

\[
= x^{(\frac{2}{3})} + x^{(\frac{1}{3})} \cdot x^{(\frac{2}{3})}
\]

Use \( x^{b} \cdot x^{d} = x^{(b+d)} \)

\[
= x^{(\frac{2}{3})} + x^{(\frac{1}{3} \cdot \frac{2}{3})}
\]

Add fractional exponents with the same base.

\[
= x^{(\frac{2}{3})} + x^{(\frac{2}{3})}
\]

Simplify a fractional exponent. \( x^{(\frac{2}{3})} = x^{\frac{1}{3}} = x \)

3. For which real values of \( x \) is \( \frac{7x}{2^{(4-x)} - 8} \) defined?

Find the values of \( x \) for which the expression is not defined.

\( \frac{7x}{2^{(4-x)} - 8} \) is not defined when \( 2^{(4-x)} - 8 = 0 \).

\( 2^{(4-x)} - 8 = 0 \) when \( 2^{(4-x)} = 8 \).

\( 2^{(4-x)} = 8 \) when \( x = 1. \) \( (2^{(4-1)} = 2^{3} = 8) \)

The expression is not defined when \( x = 1. \)

The expression is defined for all real values of \( x \) except \( x = 1. \)
MODEL ACT PROBLEM

Write the expression $\frac{\sqrt{x^3 + x}}{\sqrt{x}}$ in simplified form with no radicals and no negative exponents. ($x \neq 0$)

A. $x$
B. $x^{\frac{3}{2}}$
C. $\frac{x + 1}{x^{\frac{1}{2}}}$
D. $x + x^{\frac{1}{2}}$
E. $x^2 + \frac{1}{x}$

SOLUTION

$$\frac{\sqrt{x^3 + x}}{\sqrt{x}}$$

$$= \frac{x^{\frac{3}{2}} + x}{x^{\frac{1}{2}}}$$

$$= \frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} + \frac{x}{x^{\frac{1}{2}}}$$

$$= x^{\frac{3}{2}} + x^{\frac{1}{2}}$$

$$= x + x^{\frac{1}{2}}$$

The correct answer is D.

Practice

Simplify.

1. $\frac{x^3}{x^3} + \sqrt{x^2}$
2. $\sqrt{x^3} - \frac{x^3}{x^3}$
3. $x^{\frac{3}{2}} - \sqrt{x^3} + x^{\frac{1}{2}} + 1$
4. $\sqrt{x^3} \cdot \sqrt{x^2} + \sqrt{x^2} \cdot \sqrt{x}$
5. $x^{-5} + \sqrt{x^3} + x^{\frac{1}{2}} - \sqrt{x^{-10}}$
6. $\sqrt{x^2} + \sqrt{x^3} \cdot \sqrt{x}$
7. $x^{\frac{1}{2}} \cdot \sqrt{x^{-1}} + x^2$
8. $x^4 \cdot x^{\frac{1}{2}} + x^2 \cdot x^2$
9. $\frac{1}{x^{-6}} + \sqrt{x^{-3}} + x^{\frac{3}{2}} \cdot x$
10. $\sqrt{x^2} + \sqrt{x^3} \cdot \sqrt{x} + \sqrt{x^2}$

Identify the real values of $x$ for which each of the following expressions is defined.

11. $\frac{3x^2}{2}$
12. $\frac{\sqrt{7y - 9}}{x^2}$
13. $\frac{4k - \sqrt{65}}{6x - 2}$
14. $\frac{\sqrt{4y - 19}}{3^{x+2} - 3}$
15. $\frac{x^3 - 6}{x^3 + x^2 + 18}$

(Answers on page 361)
1. Which of the following expressions is \(\frac{4x^2y^3}{x+y} + \frac{4xy^3}{x+y}\) expressed in its simplest form?
   A. \(\frac{8x^2y}{x+y}\)
   B. \(\frac{4x^2y(x+y)}{x+y}\)
   C. \(8x^2y^2\)
   D. \(4x^2y^2\)
   E. \(4x^2y + 4xy^2\)

2. Which of the following is \(\frac{1}{\sqrt{x}} \cdot \sqrt{x} - \sqrt{x} \cdot \frac{1}{\sqrt{x}}\) in simplest form?
   F. 0
   G. 1
   H. \(1 - x\)
   J. \(\frac{1}{x^{3/2}} - x^{1/2}\)
   K. \(\frac{1}{x^{3/2}} - 1\)

3. For which real values of \(x\) is the expression \(\frac{7x}{2^{(3x-1)}}\) defined?
   A. All real values
   B. All real values except \(\frac{1}{3}\)
   C. All real values except 0
   D. All real values except 2
   E. All real values except 3

4. What are the real numbers \(x\) such that \(\frac{2x^2 + \sqrt{x}}{x^2 + x - 6}\) is defined?
   F. All real numbers
   G. All real numbers except 2
   H. All real numbers except -3
   J. All non-negative real numbers except 2
   K. All non-negative real numbers except 3

5. Which of the following is \(\frac{-2x^2 + 2y^2}{x - y}\) in simplest form? \((x - y \neq 0)\)
   A. \(-2x + 2y\)
   B. \(-2x - 2y\)
   C. \(2x - 2y\)
   D. \(x + y\)
   E. \(x - y\)

(Answers on page 361)

# Solving Quadratic Equations

Quadratic equations can be written in this standard form: \(ax^2 + bx + c = 0\) \((a \neq 0)\).
Since the largest exponent of \(x\) is 2, the equation can have at most two solutions (roots).

It may be difficult to solve a quadratic equation by factoring.

You can always use the quadratic formula to solve a quadratic equation.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

The \(a, b,\) and \(c\) in the formula are the same as the coefficients \(a, b,\) and \(c\) in the quadratic equation.
EXAMPLE

Solve. \(4x^2 - 2 = 3x\)

Write the equation in standard form. \(4x^2 - 3x - 2 = 0\)
Identify the values for \(a\), \(b\), and \(c\). \(a = 4, b = -3, c = -2\)
Substitute the values of \(a\), \(b\), and \(c\) into the quadratic formula.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-2)}}{2(4)}
\]

\[
= \frac{3 \pm \sqrt{9 + 32}}{8}
\]

\[
= \frac{3 \pm \sqrt{41}}{8}
\]

The solutions of the quadratic equation are \(\frac{3 + \sqrt{41}}{8}\) and \(\frac{3 - \sqrt{41}}{8}\).

CALCULATOR TIP

If an exact answer is not required and you have an advanced calculator, you don’t need to use the quadratic formula. Just use your calculator’s method for finding the roots of a polynomial.

MODEL ACT PROBLEMS

1. What is the sum of \(a\), \(b\), and \(c\) in the quadratic equation \(23x^2 = -13x + 6\)?

   A. \(-4\)
   B. \(4\)
   C. \(16\)
   D. \(30\)
   E. \(42\)

SOLUTION

Write the equation in standard form. \(23x^2 = -13x + 6\)
Add \(13x\) to each side. \(23x^2 + 13x = 6\)
Subtract \(6\) from each side. \(23x^2 + 13x - 6 = 0\)
Identify the values for \(a\), \(b\), and \(c\). \(a = 23, b = 13, c = -6\)
Find the sum of \(a\), \(b\), and \(c\). \(23 + 13 + (-6) = 36 - 6 = 30\)
The correct answer is D.
2. What are the solutions to the quadratic equation \( x^2 - 2x - 35 = 0 \)?

   A. 5 and 7  
   B. 5 and −7  
   C. 2 and 12  
   D. −2 and 12  
   E. −5 and 7

**SOLUTION**

Identify the values for \( a, b, \) and \( c \).

\[ a = 1, \quad b = -2, \quad c = -35 \]

Substitute the values of \( a, b, \) and \( c \) into the quadratic formula.

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ = \frac{2 \pm \sqrt{144}}{2} \]

\[ = 7 \text{ or } -5 \]

The solutions are −5 and 7.

The correct answer is E.

---

**Practice**

1. Write the quadratic formula.

2. Identify \( a, b, \) and \( c \) in the equation \( 3 - 2x^2 = -8x \).

Solve each equation.

3. \( x^2 - 3x - 4 = 0 \)

4. \( 5x^2 - 2 = 4x \)

5. \( 5x + 2 = -2x^2 \)

6. \( x^2 = x + 6 \)

7. \( 3x - 4 = -x^2 \)

8. \( 2x^2 - 5x - 3 = 0 \)

9. \( x^2 + x = 56 \)

10. \( x^2 - 18 = 7x \)

11. \( 4x^2 + 9x + 2 = 0 \)

12. \( 2x^2 + 4 = -6x \)

13. \( x^2 - 5x = -6 \)

14. \( 4x^2 - 4x - 15 = 0 \)

15. \( 26x = -3x^2 + 9 \)

16. \( 4x^2 + 25x - 21 = 0 \)

17. \( 6x^2 = 9x + 15 \)

18. Mr. Wilson’s rectangular garden has an area of 27 square feet. If the length of his garden is three times the width, what are the dimensions of the garden?

19. The area of a rectangular pool is 180 square feet. If the pool is 3 feet longer than it is wide, what are the dimensions of the pool?

20. Chan is framing an 8-inch by 10-inch picture. The area of the picture and the frame is 143 square inches. What is the width of the frame?

(Answers on page 362)
ACT-TYPE PROBLEMS

1. What is the product of \(a\), \(b\), and \(c\) in the quadratic equation \(4x^2 + 2 = -14x\)?
   - A. \(-112\)
   - B. \(-56\)
   - C. \(-28\)
   - D. \(56\)
   - E. \(112\)

2. What are the solutions to the quadratic equation \(8x^2 = -44x - 56\)?
   - F. \(-4.5\) and \(3\)
   - G. \(-3.5\) and \(1\)
   - H. \(-3.5\) and \(-2\)
   - J. \(-3\) and \(4.5\)
   - K. \(-2\) and \(3.5\)

3. What are the solutions to the quadratic equation \(2x^2 + 16x + 24 = 0\)?
   - A. \(2\) and \(6\)
   - B. \(1\) and \(12\)
   - C. \(-6\) and \(-2\)
   - D. \(-12\) and \(1\)
   - E. \(-12\) and \(-1\)

(Answers on page 362)

■ Solving Quadratic Inequalities

Factor or use the quadratic formula to solve quadratic inequalities. Use the same techniques as for quadratic equations. However, you must consider the following cases:

- If the quadratic inequality is written less than zero, then the two factors have different signs.

**Example**

Solve, \(x^2 - 9 < 0\)

\((x + 3)(x - 3) < 0\)

\((x + 3)\) and \((x - 3)\) must have different signs. That happens when:

\(x + 3 < 0\) and \(x - 3 > 0\) \hspace{20pt} OR \hspace{20pt} \(x + 3 > 0\) and \(x - 3 < 0\)

\(x < -3\) and \(x > 3\) \hspace{20pt} OR \hspace{20pt} \(x > -3\) and \(x < 3\)

This is impossible.

\(-3 < x < 3\)

The solution is all real numbers between \(-3\) and \(3\).
If the quadratic inequality is written greater than zero then the two factors must have the same sign.

**Example**

Solve.

\(x^2 - 9 > 0\)

\((x + 3)(x - 3) > 0\)

\((x + 3)\) and \((x - 3)\) must have the same sign. That happens when:

\(x + 3 > 0\) and \(x - 3 > 0\) OR \(x + 3 < 0\) and \(x - 3 < 0\)

\(x > -3\) and \(x > 3\) \(x < -3\) and \(x < 3\)

This means the number must be greater than 3.

This means the number must be less than -3.

\(x > 3\)

\(x < -3\)

The solution is all real numbers greater than 3 or less than -3.

**Model ACT Problem**

Which is the solution set of \(x^2 + 2x - 8 \geq 0\)?

A. \(x \leq 4\)

B. \(x \geq 2\)

C. \(x \leq -4\) or \(x \geq 2\)

D. \(-4 \leq x \leq 2\)

E. \(-2 \leq x \leq 4\)

**Solution**

\(x^2 + 2x - 8 \geq 0\)

\((x + 4)(x - 2) \geq 0\)

Both factors must have the same sign.

\(x + 4 \geq 0\) and \(x - 2 \geq 0\) OR \(x + 4 \leq 0\) and \(x - 2 \leq 0\)

\(x \geq -4\) and \(x \geq 2\)

\(x \leq -4\) and \(x \leq 2\)

\(x \geq 2\)

\(x \leq -4\)

The correct answer is C.

**Practice**

Write the solution set for the given inequality.

1. \(x^2 - 16 < 0\)

2. \(x^2 + 7x \leq 0\)

3. \(x^2 - 25 > 0\)

4. \(x^2 - 4x \geq 0\)

5. \(3x^2 + 10x \leq 8\)

6. \(4x^2 - 9 \geq 0\)

7. \(2x^2 - 11x + 5 \geq 0\)

8. \(x^2 > 8x + 20\)

9. \(x^2 + 27 < 12x\)

10. \(x^2 + 2x < 15\)

11. \(x^2 - 8 > 8\)

12. \(2x^2 - x - 3 > 0\)

13. \(x^2 + 1 < 2x\)

14. \(3x^2 - 12 \leq 0\)

15. \(x^2 \leq 6x - 5\)

16. \(16x^2 - 32 < 0\)

17. \(0 > -x^2 + 16\)

18. \(4x^2 + 5x \geq 0\)

19. \(x^2 + 5x \leq -4x\)

20. \(1,000,000x^2 \geq 0\)

(Answers on page 363)
1. Which is the solution set of \(x^2 - 8x + 12 < 0\)?
   A. \(x < 2\) or \(x > 6\)
   B. \(2 < x < 6\)
   C. \(x < -6\) or \(x > -2\)
   D. \(-6 < x < -2\)
   E. \(-6 < x < 2\)

2. Which quadratic inequality has the solution \(-\sqrt{3} < x < \sqrt{3}\)?
   F. \(x^2 - 3 < 0\)
   G. \(x^2 - 3 > 0\)
   H. \(x^2 - 3 \leq 0\)
   J. \(x^2 - 3 \geq 0\)
   K. \(x^2 + 3 < 0\)

3. Which is the solution set of \(x^2 + 3x - 4 > 0\)?
   A. \(-1 < x < 4\)
   B. \(-4 < x < 1\)
   C. \(x < -1\) or \(x > 4\)
   D. \(x < -4\) or \(x > 1\)
   E. \(x < -4\) or \(x > -1\)

4. Which is the solution set of \(x^2 - 3x \leq 0\)?
   F. \(x < 0\) or \(x > 3\)
   G. \(0 < x < 3\)
   H. \(0 \leq x \leq 3\)
   J. \(-3 < x < 0\)
   K. \(-3 \leq x \leq 0\)

5. Which quadratic inequality has the solution set \(-3 \leq x \leq 8\)?
   A. \(x^2 + 5x - 24 < 0\)
   B. \(x^2 - 5x + 24 \leq 0\)
   C. \(x^2 - 5x - 24 \geq 0\)
   D. \(x^2 - 5x - 24 < 0\)
   E. \(x^2 - 5x - 24 \leq 0\)

(Answers on page 363)

Complex Numbers

CALCULATOR TIP

Use a calculator that can represent complex numbers to check your work.

You will most frequently encounter complex numbers as you solve quadratic equations.

Imaginary Numbers

We call \(\sqrt{-1}\) an imaginary number. It is neither a whole number, a decimal, nor a rational number. We use the symbol \(i\) to represent this imaginary number.

\(\sqrt{-1} = i\)

The Square of \(i\)

The square of \(i\) is \(-1\).

\(i^2 = -1\)

\(24i^2 = -24\)

Standard Form

Every complex number has a standard form, \(a + bi\), where \(a\) and \(b\) are real numbers.
Complex Addition

Treat $i$ as a variable when you add complex numbers. Look at these examples.

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(3 + 4i) + (5 + 6i) = (3 + 5) + (4 + 6)i = 8 + 10i$$

Complex Multiplication

Treat $i$ as a variable when you multiply complex numbers, but remember that $i^2 = -1$. Look at these examples.

$$(a + bi)(c + di) = ac + adi + bci + bd i^2 = (ac - bd) + (ad + bc)i$$

$$(3 + 4i)(5 + 6i) = 15 + 18i + 20i + 24i^2 = (15 - 24) + (18 + 20)i = -9 + 38i$$

Complex Division

The expression $(3 + 2i) ÷ (1 + i)$ can be written as $\frac{3 - 2i}{1 + i}$. However, this is not a complex number in the form $a + bi$. We can simplify the fraction by multiplying by the complex conjugate of the denominator. The complex conjugate of $a + bi$ is $a - bi$. Look at these examples.

$$\frac{3 - 2i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{3 - 3i - 2i + 2i^2}{1 - i + i - i^2} = \frac{3 - 5i - 2}{1 + 1} = \frac{1 - 5i}{2} = \frac{1}{2} - \frac{5}{2}i$$

$$\frac{3 - 4i}{2 - 6i} \cdot \frac{2 + 6i}{2 + 6i} = \frac{6 + 18i - 8i - 24i^2}{4 + 12i - 12i - 36i^2} = \frac{6 + 10i - 24}{4 + 36} = \frac{30 + 10i}{40} = \frac{3}{4} + \frac{1}{4}i$$

Write the value of the number. Use standard form if possible.

**MODEL ACT PROBLEMS**

1. Which of the following choices represents $3 + 4i - 7 + 5i$ in standard form?
   
   A. $7i - 2i$
   B. $-4 + 9i$
   C. $-4 + 9i^2$
   D. $5i$
   E. $4 - i$

   **SOLUTION**

   Standard form for a complex number is $a + bi$. Treat $i$ as a variable when you add complex numbers.

   $$3 + 4i - 7 + 5i = (3 - 7) + (4i + 5i) = -4 + 9i$$

   The correct answer is B.

2. In simplest form, $(6 - 4i)(5 + 2i) = ?$

   A. $30 - 8i^2$
   B. $30 - 8i - 8i^2$
   C. $22 - 8i$
   D. $38 - 8i$
   E. $38 + 8i$

   **SOLUTION**

   Use FOIL.

   $$(6)(5) + (6)(2i) - (4i)(5) - (4i)(2i)$$

   Simplify.

   $$30 + 12i - 20i - 8i^2$$

   Combine like terms.

   $$30 - 8i^2 + 12i - 20i$$

   Remember $i^2 = -1$.

   $$38 - 8i$$

   The correct answer is J.
Practice

1. \(18i^2\)  
2. \(i^3\)  
3. \(\sqrt{-49}\)  
4. \(\sqrt{-50}\)  
5. \(\sqrt{-48}\)  

Write in standard form.

6. \(18 - 6 + 5i\)  
7. \(3 \times 9 + 3i\)  
8. \(4i - 16 - 5i\)  
9. \(2 + 7i - 6 + 5i\)  
10. \(7(6 - \sqrt{-1})\)  

Add.

11. \((2 + 9i) + (3 + 7i)\)  
12. \((12 + 2i) + (7 + 3i)\)  
13. \((13 + 9i) + (-21 + 7i)\)  
14. \((12.5i^2 + 3i) + (-8 + 6i)\)  
15. \(\left(\frac{1}{2} + \frac{3}{4}i\right) + \left(\frac{1}{4} + \frac{1}{8}i\right)\)  

Multiply or divide. Express answers in \(a + bi\) form.

16. \((2 + 9i)(3 + 7i)\)  
17. \((12 + 2i)(7 + 3i)\)  
18. \((13 + 9i)(-4 + i)\)  
19. \((0.5 + 3i)(-8 + 6i)\)  
20. \(\left(\frac{1}{2} + \frac{3}{4}i\right) \times \left(\frac{1}{4} + \frac{2}{3}i\right)\)  
21. \(4 \div i\)  
22. \(\frac{5}{-2 + 6i}\)  
23. \((2 - 0.5i) \div (1 + 0.5i)\)  
24. \(\frac{-7 + 2i}{9 + 5i}\)  

(Answers on page 363)

ACT-TYPE PROBLEMS

1. In standard form, \(-3(6 - \sqrt{-25}) = \) ?
   A. \(-18 + 15i\)  
   B. \(-18 - 15i\)  
   C. \(-18 - 5i\)  
   D. \(-3\)  
   E. \(-3i\)  

2. Find the sum of \(5 - 2i\) and \(-3 + 7i\).
   F. \(8 - 9i\)  
   G. \(-2 + 9i\)  
   H. \(2 - 5i\)  
   J. \(2 + 5i\)  
   K. \(2 + 5i^2\)  

3. Use the quadratic formula to find the roots of \(x^2 + 4 = 0\).
   A. \(-2\) only  
   B. \(-2\) and \(2\)  
   C. \(4 + 2i\) and \(4 - 2i\)  
   D. \(-8i\) and \(8i\)  
   E. \(-2i\) and \(2i\)  

4. Find the quadratic equation whose roots are \(3i\) and \(-3i\).
   F. \(x^2 - 9 = 0\)  
   G. \(x^2 + 9 = 0\)  
   H. \(x^2 - 6ix + 9 = 0\)  
   J. \(x^2 - 6ix - 9 = 0\)  
   K. \(x^2 + 6ix - 9 = 0\)  

5. Use the quadratic formula to find the roots of \(x^2 - 6x + 10 = 0\).
   A. \(2\) and \(4\)  
   B. \(2\) and \(5\)  
   C. \(-3i\) and \(3i\)  
   D. \(3 + i\) and \(3 - i\)  
   E. \(3 + \sqrt{19}\) and \(3 - \sqrt{19}\)  

(Answers on page 363)
Patterns, Sequences, and Modeling

Patterns

Pascal’s triangle is among the most famous patterns in mathematics.

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
```

Each term in the triangle is the sum of the two terms immediately above it to the left and to the right. This triangle can be used to model probability events, and each row shows the coefficients of a power of the binomial \( x + 1 \). \((x + 1)^3 = x^3 + 3x^2 + 3x + 1.\)

Sequences

A sequence is a list of numbers that follows a pattern. Sequences frequently model events. The square and triangular numbers below form sequences.

**Triangular Numbers**

```
1  3  6 10
```

**Square Numbers**

```
1  4  9 16
```

You can find patterns within sequences. Notice that the sum of two consecutive triangular numbers yields a square number.

The Fibonacci sequence is the most famous mathematics sequence. The first eleven terms of the Fibonacci sequence are shown below. The pattern starts with 1 and then each term is the sum of the two previous terms.

```
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 ...
```

Fibonacci numbers model many natural occurrences, including the swirls found in pine cones and the number of petals on sunflowers. The quotient of two consecutive Fibonacci numbers approximates the golden ratio (1.6). This ratio is found often in nature and in architecture.

**Arithmetic Sequence**

In an arithmetic sequence, consecutive terms differ by the same amount, called the common difference.
**Example**

3, 5, 7, 9, 11, … is an arithmetic sequence with a common difference of 2.

10, 7, 4, 1, −2, … is an arithmetic sequence with a common difference of −3.

The general term of an arithmetic sequence is given by the formula:

\[a_n = a_1 + (n - 1)d\]

\(a_n\) is the \(n\)th term.

\(a_1\) is the first term.

\(n\) is the position in the sequence.

\(d\) is the common difference.

**Geometric Sequence**

In a geometric sequence, consecutive terms vary from each other by a factor called the common ratio, \(r\).

\[a_n = a_1r^{n-1}\]

**Example**

3, 6, 12, 24, 48, … is a geometric sequence with a common ratio of 2.

36, 12, 4, \(\frac{4}{9}\), \(\frac{4}{9}\), \(\frac{4}{9}\), … is a geometric sequence with a common ratio of \(\frac{1}{3}\).

1, −4, 16, −64, 256, … is a geometric sequence with a common ratio of −4.

The general term of a geometric sequence is given by the formula:

\[a_n = a_1r^{n-1}\]

**Model ACT Problem**

Greg went to the duck pond every day for a week. The table shows the number of ducks Greg saw each day.

<table>
<thead>
<tr>
<th>Day 1</th>
<th>Day 2</th>
<th>Day 3</th>
<th>Day 4</th>
<th>Day 5</th>
<th>Day 6</th>
<th>Day 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>128</td>
</tr>
</tbody>
</table>

If this pattern remains the same, how many ducks will Greg see on the 11th consecutive day he goes to the duck pond?

A. 136 ducks  
B. 144 ducks  
C. 512 ducks  
D. 1,024 ducks  
E. 2,048 ducks

**Solution**

The problem shows a geometric sequence with a common ratio of 2. You can solve the problem by just multiplying by 2 until you reach the eleventh term.

You can also use the formula for the \(n\)th term of a geometric sequence.

\[a_n = a_1r^{n-1}\]

\[a_{11} = 2(2)^{11-1} = 2(2)^{10} = 2 \times 1,024 = 2,048\]

Use your calculator.

The correct answer is E.
Write the next three terms in each sequence.

1. 5, 5, 10, 15, 25, 40, 65, ...
2. 1, 3, 5, 7, 9, ...
3. 97, 86, 75, 64, 53, ...
4. 1, 3, 4, 7, 11, 18, 29, 47, ...
5. 3, 8, 13, 18, 23, ...
6. 1, 8, 27, 64, 125, ...
7. 2,000; 1,000; 500; 250; 125; ...
8. 2, 6, 18, 54, 162, 486, ...
9. 39, 31, 23, 15, 7, ...
10. 2, 3, 5, 7, 11, 13, 17, 19, ...
11. 2, 4, 6, 8, 10, ...
12. 1, 2, 4, 8, 16, 32, ...
13. 3, 6, 12, 24, 48, ...
14. 1, 4, 9, 16, 25, ...
15. \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots \)
16. 15, 11, 7, 3, 
17. 2, 3, 5, 7, 11, ...
18. \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots \)
19. 1, 3, 7, 15, 31, ...
20. \( \frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \ldots \)

(Answers on page 364)

ACT-TYPE PROBLEMS

1. Gary swims every week. The table shows the total amount of time Gary swims in five consecutive weeks. If this pattern continues, how long will Gary swim in the 6th week?

<table>
<thead>
<tr>
<th>Week</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>75 min</td>
</tr>
<tr>
<td>2</td>
<td>84 min</td>
</tr>
<tr>
<td>3</td>
<td>93 min</td>
</tr>
<tr>
<td>4</td>
<td>102 min</td>
</tr>
<tr>
<td>5</td>
<td>111 min</td>
</tr>
</tbody>
</table>

A. 122 minutes  
B. 120 minutes  
C. 118 minutes  
D. 116 minutes  
E. 114 minutes

2. The first term in a geometric sequence is 3, and the common factor is 2. Which of the following shows the first 5 terms in the sequence?

F. 3, 5, 7, 8, 9  
G. 3, 6, 12, 24, 48  
H. 2, 5, 8, 11, 14  
J. 2, 6, 18, 54, 162  
K. 3, 18; 108; 648; 3,888

3. Which of the following choices displays the seventh term in the sequence below?

4 32 256 2,048

A. 131,072  
B. 442,368  
C. 838,860  
D. 1,048,576  
E. 8,388,608

4. 81 is the ninth term in which of the following sequences?

F. 1, 3, 6, 10, 15, ...  
G. 11, 21, 31, 41, 51, ...  
H. 1, 4, 9, 16, 25, ...  
J. 8, 16, 24, 32, 40, ...  
K. 5, 10, 15, 20, 25, ...

5. What are the next three numbers in the sequence
1, 2, 3, 5, 7, 11, ... ?

A. 24, 36, 52  
B. 32, 64, 128  
C. 48, 96, 192  
D. 32, 48, 64  
E. 48, 64, 136

(Answers on page 364)

Matrices

A matrix is a rectangular array of numbers or variables. The entries in a matrix are called elements. Examples of matrices are shown below.

A. \[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6
\end{bmatrix}
\]
B. \[
\begin{bmatrix}
z & y \\
x & w
\end{bmatrix}
\]
C. \[
\begin{bmatrix}
10 & 3 & 5 & 9
\end{bmatrix}
\]
D. \[
\begin{bmatrix}
-6 & 5 & 123 \\
19 & -51 & -1.8 \\
23 & 2 & -8
\end{bmatrix}
\]

180 Mathematics
The horizontal entries are called rows, while the vertical entries are called columns. Notice that there are the same number of elements in each row, and the same number of elements in each column.

**Dimension**

The dimension of a matrix is the number of rows followed by the number of columns. Here are the dimensions of each matrix shown above:

- A. \(2 \times 3\)
- B. \(3 \times 2\)
- C. \(1 \times 4\)
- D. \(3 \times 3\)

**Scalar**

Scalar is just another name for a number.

**Matrix Arithmetic**

You are most likely to encounter matrix addition and scalar multiplication on the ACT.

**Matrix Addition**

You may add matrices that have the same dimension. Just add the corresponding elements of the matrices to form a new matrix. Look at this example.

\[
\begin{bmatrix}
3 & 8 \\
-9 & 6 \\
13 & -6
\end{bmatrix}
+ 
\begin{bmatrix}
6 & -5 \\
-3 & -14 \\
0 & 9
\end{bmatrix}
= 
\begin{bmatrix}
3 + 6 & 8 + (-5) \\
-9 + (-3) & 6 + (-14) \\
13 + 0 & -6 + 9
\end{bmatrix}
= 
\begin{bmatrix}
9 & 3 \\
-12 & -8 \\
13 & 3
\end{bmatrix}
\]

**Scalar Multiplication**

Multiply each element in the matrix by a scalar (number). Look at this example.

\[
7 \begin{bmatrix}
3 & 8 & -9 & 6 \\
12 & -7 & 5 & 0
\end{bmatrix}
= 
\begin{bmatrix}
7 \times 3 & 7 \times 8 & 7 \times (-9) & 7 \times 6 \\
7 \times 12 & 7 \times (-7) & 7 \times 5 & 7 \times 0
\end{bmatrix}
= 
\begin{bmatrix}
21 & 56 & -63 & 42 \\
84 & -49 & 35 & 0
\end{bmatrix}
\]

**Matrix Multiplication**

Multiplying two matrices is more complicated than multiplying a matrix by a scalar. Look at this example.

\[
\begin{bmatrix}
5 & -4 & 2 \\
1 & 3 & -2
\end{bmatrix}
\times 
\begin{bmatrix}
3 & -1 \\
0 & 5 \\
-2 & 0
\end{bmatrix}
\]

Step 1. Find the dimension of the product. This step is easy.
Write the dimension of the first matrix: \(2 \times 3\)
Write the dimension of the second matrix: \(3 \times 2\)
The product will have the same number of rows as the first matrix and the same number of columns as the second matrix.

\[
\begin{array}{c|c}
1\text{st matrix} & 2\text{nd matrix} \\
2 \times 3 & 3 \times 2
\end{array}
\]

**Dimension of the product**

The dimension of the product will be \(2 \times 2\).
Step 2. Multiply the matrices. For each element of the product, multiply a row in the first matrix by a column in the second matrix and add the products of the elements. For example, to find the top left element, multiply the first row in the first matrix by the first column in the second. This is highlighted below:

\[
\begin{bmatrix}
5 & -4 & 2 \\
1 & 3 & -2
\end{bmatrix}
\times
\begin{bmatrix}
3 & -1 \\
0 & 5 \\
-2 & 0
\end{bmatrix} =
\begin{bmatrix}
5(3) - 4(0) + 2(-2) & ? \\
? & ?
\end{bmatrix} =
\begin{bmatrix}
11 & ? \\
? & ?
\end{bmatrix}
\]

Repeat this process for all the elements in the product:

\[
\begin{bmatrix}
5 & -4 & 2 \\
1 & 3 & -2
\end{bmatrix}
\times
\begin{bmatrix}
3 & -1 \\
0 & 5 \\
-2 & 0
\end{bmatrix} =
\begin{bmatrix}
11 & 5(-1) - 4(5) + 2(0) \\
1(3) + 3(0) - 2(-2) & 1(-1) + 3(5) - 2(0)
\end{bmatrix} =
\begin{bmatrix}
11 & -25 \\
7 & 14
\end{bmatrix}
\]

**Problem Solving With Matrices**

You can use matrix multiplication to solve one type of problem that may appear on your test. You may see a problem like this on the ACT:

**EXAMPLE**

Bob and Liz purchased bags of candy, Bags A, B, and C. Bob bought six of Bag A, four of Bag B, and nine of Bag C. Liz bought four of Bag A, eight of Bag B, and five of Bag C. Bags of candy A cost $3, bags of candy B cost $6, and bags of Candy C cost $8. What was the total cost of all the candy in Bags A, B, and C that Bob and Liz bought?

Here's how to solve it using matrix multiplication.

Write a matrix for how many bags of candy each person bought.

<table>
<thead>
<tr>
<th>Number of Bags</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Bob</td>
<td>6</td>
</tr>
<tr>
<td>Liz</td>
<td>4</td>
</tr>
</tbody>
</table>

Multiply:

\[
\begin{bmatrix}
6 & 4 & 9 \\
4 & 8 & 5
\end{bmatrix}
\times
\begin{bmatrix}
$3 \\
$6 \\
$8
\end{bmatrix} =
\begin{bmatrix}
6 \times $3 + 4 \times $6 + 9 \times $8 \\
4 \times $3 + 8 \times $6 + 5 \times $8
\end{bmatrix} =
\begin{bmatrix}
$18 + $24 + $72 \\
$12 + $48 + $40
\end{bmatrix} =
\begin{bmatrix}
$114 \\
$100
\end{bmatrix}
\]
The top entry in the final matrix shows the cost of Bob's candy.
The bottom entry shows the cost of Liz's candy.

Add the entries to find the total cost.

$114 + $100 = $214

Note that you do not need matrices to solve this problem. You can just think the problem through.

Candy A: 6 + 4 = 10 bags, Candy B: 4 + 8 = 12 bags, Candy C: 9 + 5 = 14 bags.

Then multiply by the cost:

$10 \times $3 + 12 \times $6 + 14 \times $8 = $30 + $72 + $112 = $214

**MODEL ACT PROBLEM**

\[
\begin{bmatrix}
8 & -8 \\
0 & -2
\end{bmatrix}
+ 
\begin{bmatrix}
-2 & -2 \\
-2 & -2
\end{bmatrix} = ?
\]

**SOLUTION**

Add the corresponding elements.

\[
\begin{bmatrix}
8 & -8 \\
0 & -2
\end{bmatrix}
+ 
\begin{bmatrix}
-2 & -2 \\
-2 & -2
\end{bmatrix} =
\begin{bmatrix}
8 + (-2) & -8 + (-2) \\
0 + (-2) & -2 + (-2)
\end{bmatrix}
= \begin{bmatrix}
6 & -10 \\
-2 & -4
\end{bmatrix}
\]

The correct answer is B.

**Practice**

Use the matrices shown below to answer questions 1–15.

A. \[
\begin{bmatrix}
-1 & 1 & -4 \\
9 & -7 & 0 \\
12 & -9 & 2
\end{bmatrix}
\]

B. \[
\begin{bmatrix}
3 & -8 \\
6 & 12 \\
5 & 10
\end{bmatrix}
\]

C. \[
\begin{bmatrix}
10 & 3 & 5 \\
-3 & 6 & 2
\end{bmatrix}
\]

D. \[
\begin{bmatrix}
-6 & 5 & 12 \\
19 & -1 & -2 \\
23 & 2 & -8
\end{bmatrix}
\]

E. \[
\begin{bmatrix}
5 & -9 & 12 \\
-3 & 7 & 8
\end{bmatrix}
\]

F. \[
\begin{bmatrix}
9 & 18 & -12 \\
0 & 3 & -6 \\
-2 & 8 & 9
\end{bmatrix}
\]

G. \[
\begin{bmatrix}
7 & 2 & -5 \\
12 & 3 & 5
\end{bmatrix}
\]

Write the dimension of the matrix.


Add these matrices.

Find the scalar product.

11. $-1 \times G$  
12. $2 \times F$  
13. $7 \times C$  
14. $-2 \times B$  
15. $0.5 \times H$

Subtract the matrices.

16. $C - E$  
17. $A - F$  
18. $D - F$  
19. $F - A$  
20. $D - H$

(Answers on page 364)

### ACT-TYPE PROBLEMS

1. What are the dimensions of this matrix?
   
   \[
   \begin{bmatrix}
   5 & 2 & -3 \\
   4 & -7 & 0
   \end{bmatrix}
   \]
   
   A. $1 \times 3$  
   B. $3 \times 1$  
   C. $3 \times 0$  
   D. $5 \times 2 \times 3$  
   E. $3$

2. \[
\begin{bmatrix}
5 & 2 & -3 \\
4 & -7 & 0
\end{bmatrix} + \begin{bmatrix}
5 & 4 \\
2 & -7
\end{bmatrix} = ?
\]
   
   F. \[
\begin{bmatrix}
10 & 4 & -6 \\
8 & -14 & 0
\end{bmatrix}
\]
   
   G. \[
\begin{bmatrix}
10 & 8 \\
4 & -14 \\
-6 & 0
\end{bmatrix}
\]
   
   H. \[
\begin{bmatrix}
10 & 6 & -1 \\
-3 & -10 & 0
\end{bmatrix}
\]
   
   J. \[
\begin{bmatrix}
10 & -3 \\
6 & -10 \\
-1 & 0
\end{bmatrix}
\]
   
   K. The two matrices cannot be added.

3. Multiply.

\[
\begin{bmatrix}
-3 & 4 & -7 & -1/2
\end{bmatrix}
\]
   
   A. \[
\begin{bmatrix}
-12 & 21 & -3/2
\end{bmatrix}
\]
   
   B. \[
\begin{bmatrix}
-12 \\
21
\end{bmatrix}
\]
   
   C. \[
\begin{bmatrix}
-12 & 21 & 3/2
\end{bmatrix}
\]
   
   D. \[
\begin{bmatrix}
1 & -10 & -3
\end{bmatrix}
\]

4. \[
\begin{bmatrix}
2 & 3 & 4 \\
-2 & -3 & -4
\end{bmatrix} + \begin{bmatrix}
-2 & -3 & -4
\end{bmatrix} = ?
\]
   
   F. \[
\begin{bmatrix}
2 & 3 & 4 \\
-2 & -3 & -4
\end{bmatrix}
\]
   
   G. \[
\begin{bmatrix}
-4 & -6 & -8
\end{bmatrix}
\]
   
   H. \[
\begin{bmatrix}
-4 & -9 & -16
\end{bmatrix}
\]
   
   J. \[
\begin{bmatrix}
0 & 0 & 0
\end{bmatrix}
\]
   
   K. \[
\begin{bmatrix}
0 & 0 & 0
\end{bmatrix}
\]

5. Subtract.

\[
\begin{bmatrix}
1 & 3 \\
5 & 7 \\
9 & 11
\end{bmatrix} - \begin{bmatrix}
-1 & -3 \\
-5 & -7 \\
-9 & -11
\end{bmatrix} = ?
\]
   
   A. \[
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]
   
   B. \[
\begin{bmatrix}
2 & 6 \\
10 & 14 \\
18 & 22
\end{bmatrix}
\]
   
   C. \[
\begin{bmatrix}
-2 & -6 \\
-10 & -14 \\
-18 & -22
\end{bmatrix}
\]
   
   D. \[
\begin{bmatrix}
8 \\
24 \\
40
\end{bmatrix}
\]
   
   E. \[
\begin{bmatrix}
1 & 3 & -1 & -3 \\
5 & 7 & -5 & -7 \\
9 & 11 & -9 & -11
\end{bmatrix}
\]
6. A car dealership had a special sale on Wednesday and another on Thursday. Convertibles (C), midsized cars (M), and SUVs (S) were sold on Wednesday and Thursday. The matrices below show the number of each type of car sold for each day and the bonus the dealer received for each car sold from the manufacturer. What total bonus did the dealership receive from the manufacturer for the convertibles, midsized cars, and SUVs sold on those two sale days?

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>M</th>
<th>S</th>
<th>Bonus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wednesday</td>
<td>24</td>
<td>38</td>
<td>13</td>
<td>$200</td>
</tr>
<tr>
<td>Thursday</td>
<td>32</td>
<td>52</td>
<td>11</td>
<td>$100</td>
</tr>
</tbody>
</table>

F. $10,200  
G. $12,500  
H. $14,900  
J. $27,400  
K. $30,800

7. \[ \begin{bmatrix} 450 & 700 & 900 \end{bmatrix} \times \begin{bmatrix} 28 & 81 & 41 \\ 30 & 36 & 32 \\ 51 & 25 & 29 \end{bmatrix} = ? \]

A. \[ \begin{bmatrix} 79,500 & 84,150 & 66,950 \end{bmatrix} \]
B. \[ \begin{bmatrix} 49,050 & 99,400 & 91,800 \end{bmatrix} \]
C. \[ \begin{bmatrix} 230,600 \end{bmatrix} \]
D. \[ \begin{bmatrix} 67,500 \\ 68,600 \\ 94,500 \end{bmatrix} \]
E. \[ \begin{bmatrix} 79,500 \\ 84,150 \\ 66,950 \end{bmatrix} \]

8. The Amsco designer shirts were hot sellers. Amsco sells three types of T-shirts: regular T-shirts (R), T-shirts with collars (C), and long sleeve T-shirts (L). The T-shirts are sold in three stores located in New York, Chicago, and Detroit. The matrix below gives the number of shirts sold at each store on opening day.

\[
\begin{array}{ccc}
\text{New York} & R & C & L \\
120 & 75 & 35 \\
\text{Chicago} & 125 & 60 & 25 \\
\text{Detroit} & 90 & 35 & 40 \\
\end{array}
\]

The price of each T-shirt is shown in the following matrix.

\[
\begin{array}{c}
\text{R} \\
\text{C} \\
\text{L} \\
\end{array}
\begin{array}{c}
\$22.50 \\
\$29.00 \\
\$36.50 \\
\end{array}
\]

Given these matrices, what were the total sales for opening day?

F. $16,117.50  
G. $20,247.50  
H. $25,975  
J. $27,264  
K. $67,145

(Answers on page 364)
Section IV

Science Reasoning

Chapter 18 \ The Four-Step Approach to Taking the ACT Science Reasoning Test 477
Chapter 19 \ Data Representation 484
Chapter 20 \ Research Summaries 495
Chapter 21 \ Conflicting Viewpoints 503
Chapter 22 \ Diagnostic Science Reasoning ACT: Model Science Reasoning ACT I 511
The Four-Step Approach to Taking the **ACT** Science Reasoning Test

The ACT Science Reasoning Test is not a test about science facts. It is a test of your ability to read and understand science materials. The more experience you have reading science, the better. However, you do not need advanced science courses to be successful. This chapter shows you how to do your absolute best on the ACT Science Reasoning Test.

In the first part of this chapter, you’ll read about the passage types and question types on the ACT. Then you’ll learn about a four-step approach to tackling the passages on the Science Reasoning Test and actually see the steps applied to a practice passage. Using the four steps will help you answer the questions more systematically, faster, and with greater accuracy. In Chapters 19, and 20, and 21, you’ll take a closer look at each of the three types of passages. These chapters will give you specific helpful tips—and give you a chance to practice using the four steps.

### Passage and Question Types

There are three types of passages and three levels of questions on the Science Reasoning Test.

**Passage Types.** There are seven passages on the test. These passages may be about biology, chemistry, physics, astronomy, geology, or meteorology. However, there are just three types of passages. The following capsule descriptions of the three passage types will give you an idea of what to expect.

**Data Representation.** Data representation passages include graphs, tables, diagrams, charts, figures, and illustrations. You might see a traditional bar, line, or circle graph, or you might see an illustration showing the labeled skeleton of the human hand. You might see a graph that shows the relationship between two variables, or you might see a tree diagram that describes how certain traits were inherited. In each case, you will be able to answer the question from the information given.

Questions for data representation passages typically ask you to read data, interpret data, or explain the science that underlies the represented data.

**Research Summary.** Research summary passages may supply the designs of experiments or give a summary of experimental results. These passages may also include graphs, tables, diagrams, charts, figures, and illustrations that show experimental results.

Questions for research summary passages typically ask you about the following:

1. Appropriateness of the experimental design
2. The impact of modifications in the design
3. The scientific concepts reflected in the experiment
4. The relationship between the experimental data and the concepts
5. The meaning of the results or the implications for future research
Conflicting Viewpoints. Conflicting viewpoints passages present two or more views that are inconsistent with one another. You are never asked to determine which viewpoint is correct. In fact, one or more of the viewpoints presented may be obviously incorrect. These passages may also include graphs, tables, diagrams, charts, figures, and illustrations.

Questions for conflicting viewpoints passages typically ask you about the following:

1. Scientific ideas or assumptions discussed in the passages
2. The similarities or differences among the viewpoints
3. Whether certain results or facts are consistent with one of the viewpoints
4. Which diagram best illustrates one of the viewpoints

Question Types. The questions on the ACT Science Reasoning Test address three different levels of thinking or reasoning.

Understanding. This type of question asks you to find information or use it at a basic level. Often, the answers can be found in the passage. Sometimes, you may have to make an inference, extrapolate data from a graph, or grasp the concepts underlying a passage.

Analysis. This type of question requires you to draw a conclusion based on the information in the passage. You may also be asked to explain experimental results or describe the relationship among data or information.

Generalization. These questions require you to go beyond the information given. To answer a generalization question, you need to apply the information in the passage to new situations or circumstances.

The Four Steps

Use these four steps as you read science reasoning passages and answer questions. Use step 1 just once for each passage. Then use step 2, step 3, and step 4 for each question.

1. Skim the passage and look over any charts and graphs. Identify the passage as data representation, research summary, or conflicting viewpoints.
2. Read the questions and all of the answers.
3. Eliminate obviously incorrect answers.
4. Choose the correct answer from the remaining choices.

Step 1. Skim and identify the passage. Do not read the passage carefully at first! You don’t want to waste time reading material you will not need to answer the question. You should be able to complete this step in less than a minute.

Skim the passage to find what each paragraph in the passage is all about. Only read the first and last sentences. You don’t want to know the details yet. Look over any charts and diagrams just enough to get an idea of what they are about. Don’t try to read or interpret the data just yet. You just want to know what sort of data is available. Identify the passage as data representation, research summary, or conflicting viewpoints.

Step 2. Read the question and all the answers. Read each question and all the answer choices. Be sure you are clear about what the question is asking. You want to answer the question on the test, not some other question.

Step 3. Eliminate obviously incorrect answers. Cross off any answers you’re sure are incorrect. Eliminating incorrect answers is a big help in determining the correct answer.
Step 4. Choose the correct answer from the remaining choices. Choose the answer that is most correct. Go back to the passage to confirm your choice. If you don’t know the correct answer, guess. There is no penalty for guessing. Never leave an answer choice blank.

Let’s apply these four steps to the following sample passage and its questions. This passage and questions are as difficult as the most challenging material you’ll encounter on the actual ACT.

ACT Sample Passage With Answers Explained

Scientists are interested in evaluating water quality in areas used for farming (cropped areas) and in other regions of the research area. Cropped areas are heavily fertilized with nitrogen fertilizers. Water was sampled at four different types of wells: field wells, upgradient wells, wetland wells, and off-site wells. Water flows horizontally past field wells. Water flows down from upgradient wells. Wetland wells are in a marsh area. Off-site wells are outside the research area. Water was collected each month for a year.

Figure 1 shows the research area and part of the surrounding area.

Figure 1
Figure 2 shows the results of the research. Cations are positively charged ions. Anions are negatively charged ions.

1. Figure 1 shows that the area of the experimental region is about:
   A. 10 kilometers.
   B. 0.25 square mile.
   C. 0.14 square kilometer.
   D. 1 square mile.

2. Which of the following best explains the reason why nitrate concentrations are highest in field wells?
   F. The water flows down from the upgradient wells, taking most of the nitrate with it.
   G. Field wells are located in cropped areas or in the runoff from cropped areas.
   H. Water flows horizontally past field wells, so more nitrate adheres to the soil and is left behind.
   J. Field wells have the highest concentration of chloride.

3. Which of the following statements best represents the results of this investigation?
   A. The maximum amount of bicarbonate in upgradient wells is about the same as the maximum amount of sulfate in upgradient wells.
   B. The median amount of calcium in off-site wells is about twice the maximum amount of nitrate in off-site wells.
   C. The median amount of calcium in upgradient wells is about twice the minimum amount of magnesium in upgradient wells.
   D. The maximum amount of chloride in field wells is about twice the maximum amount of chloride in off-site wells.

Figure 2